



I Semester B.A./B.Sc. Examination, April/May 2021
(CBCS) (2020 – 21 and Onwards) (Fresh)
Paper – I : MATHEMATICS

Time : 3 Hours

Max. Marks : 70

Instruction : Answer *all* questions.

PART – A

I. Answer **any five** questions.

(5×2=10)

a) Reduce the matrix $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ to echelon form.

b) Find the eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

c) Find the n^{th} derivative of $\sin^2 x$.

d) If $z = x^3 - 4x^2y + 5y^2$, find $\frac{\partial^2 z}{\partial x \partial y}$.

e) Evaluate $\int_0^{\pi/2} \cos^8 x \, dx$.

f) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$.

g) Find the equation of the sphere whose centre is $(3, -1, 4)$ and which passes through the point $(1, -2, 0)$.

h) Show that the spheres $x^2 + y^2 + z^2 + 6y + 14z + 28 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 24 = 0$ cut orthogonally.



PART – B

II. Answer **any three** questions.

(5×3=15)

a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by reducing it to normal form.

P.T.O.



b) Solve the system of equations :

$$x_1 + 3x_2 + 2x_3 = 0$$

$$2x_1 - x_2 + 3x_3 = 0$$

$$3x_1 - 5x_2 + 4x_3 = 0$$

$$x_1 + 17x_2 + 4x_3 = 0.$$

c) Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and solve them.

d) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

e) Using Cayley Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

PART – C

III. Answer **any three** questions.

(5×3=15)

a) Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$.

b) If $x = \sin t$ and $y = \cos pt$ then prove that $(1 - x^2) y_{n+2} - (2n+1)xy_{n+1} - (n^2 - p^2) y_n = 0$.

c) State and prove Euler's theorem for homogeneous function of two variables.

d) If $z = \sin\left(\frac{x}{y}\right)$ and $x^2 + y^2 = a^2$, find $\frac{dz}{dx}$.

e) If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

PART – D

IV. Answer **any two** questions.

(5×2=10)

a) Obtain the reduction formula for $\int \sec^n x \, dx$, where n is a positive integer.

b) Evaluate $\int_0^\pi x \sin^6 x \, dx$.

c) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} \, dx$, where a is a parameter, using differentiation under integral sign.



PART – E

V. Answer **any two** questions.

(5×2=10)

- a) Find the equation of the tangent plane to the sphere, $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at $(1, 2, 3)$.
- b) Find the equation of the right circular cone generated by revolving the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ about the line $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$.
- c) Find the equation of a right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has the direction cosines proportional to $2, -3, 6$.

PART – F

VI. Answer **any two** questions.

(5×2=10)

- a) If $y = (t^2 + 1)^2$ is the displacement of a particle. Find its velocity and acceleration at $t = 2$.
 - b) If 15 kgs of commodity A and 17 kgs of commodity B together cost Rs. 241 and 25 kgs of commodity A and 13 kgs of commodity B together costs Rs. 279. Find the price of each per kg by using row reduced echelon form.
 - c) If a metallic ball of radius 3 units is enclosed within a cylindrical pipe. Find the equation of the cylinder, if its axis is $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.
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